

ABSTRACT

LUNAR TRANSFER TRAJECTORY DESIGN AND THE FOUR-BODY PROBLEM

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Condensed Abstract

The existence of a ballistic trajectory from the Earth to orbit about the Moon was long considered to be impossible based on analysis of the three-body problem. In 1990 a ballistic trajectory from the Earth to lunar orbit was discovered while analyzing a plan to salvage the Muses A (Hiten) spacecraft. This trajectory utilized the Sun's gravity in conjunction with the Earth and Moon's gravity and was thus the first example of a practical four-body trajectory design. This paper presents a review of lunar transfer trajectories that go beyond three-body theory and the Jacobi integral. These include Hiten, Lunar A and the Genesis return trajectory from the vicinity of the Moon to Earth. It is shown that these trajectories may be analyzed by piecing together segments where three-body motion dominates.

Extended Abstract

An early investigation of flight to the Moon by V. A. Egorov in 1958 identified several problems. These included hitting the Moon, circumnavigation of the Moon with a return to Earth at a flat entry angle, using the Moon's gravity for assist in reaching the planets and the possibility of the Moon capturing a projectile launched from the Earth. Based on consideration of the three-body problem and its associated Jacoby integral, solutions can be demonstrated for these problems with the exception of the Moon capturing a projectile launched from Earth. For the problem of lunar capture, Egorov concluded that the Moon couldn't possibly capture a projectile launched from the Earth on the first circuit of the trajectory no matter what initial conditions are specified. This conclusion was based on analysis of the three-body problem and did not consider the Sun's gravity.

The first example of a ballistic trajectory of a spacecraft launched from the Earth into orbit about the Moon was discovered in 1990 while analyzing a plan to salvage the Muses A (Hiten) spacecraft in a highly eccentric orbit about the Earth. The key to the discovery was the utilization of the Sun's gravity to affect the transfer to a lunar capture orbit. The result was a numerical solution to the restricted four-body problem of the Earth, Moon, Sun and a point mass spacecraft. This paper presents a review of lunar transfer trajectories that require analysis that goes beyond that provided by three-body theory and the Jacobi integral. Examples are Hiten, Lunar A, and the Genesis return trajectory from the vicinity of the Moon to Earth. These trajectory designs cannot be fully explained or analyzed using three-body theory and the Jacobi integral. As is the

case for the three-body problem, a complete analytic solution of the four-body problem has not been obtained. Furthermore, an integral relationship similar to the Jacobi integral has not been found for the four-body problem and the prospects for finding such an integral are dim. Current theories, such as Weak Stability Theory, are explanatory and not predictive and thus cannot be effectively used for design of trajectories that require a simultaneous four-body solution.

In the absence of a predictive four-body theory, the trajectory designer may use the existing solution of the two-body problem and the Jacobi integral to piece together trajectory segments and achieve the desired result. Indeed, most lunar transfer trajectory designs are obtained by patching together conic orbits where the Earth's gravity dominates to conic orbits where the Moon's gravity dominates. By extension, the trajectory segment dominated by the Earth, Moon and spacecraft Jacobi integral may be pieced together with the trajectory segment dominated by the Sun, Earth and spacecraft Jacobi integral to obtain continuous ballistic trajectories that connect Earth departure or arrival with capture orbits about the Moon or the nearby Lagrange points.